

Groth16 Proof Aggregation: Cryptography and Implementation Review

Protocol Labs

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Executive Summary



Synopsis

During April 2021, Protocol Labs engaged NCC Group's Cryptography Services team to conduct a cryptography and implementation review of the Groth16 proof aggregation functionality in the **bel1person** and two other related GitHub repositories. This code utilizes inner product arguments to efficiently aggregate existing Groth16 proofs while re-using existing powers of tau ceremony transcripts. Full source code access was provided with support over Slack. The project concluded with a brief retest several weeks after the initial review.

Scope

NCC Group's evaluation included the following primary materials:

- The bellperson GitHub repository, commits d4120 78 and 854f254 on branch feat-ipp2, containing source code in src/groth16/aggregate/*
- The filecoin-ffi GitHub repository, commit c5 e646e on branch feat-aggregation2, containing source code in rust/src/proofs/api.rs with the primary functions fil_aggregate_seal_proofs() and fil_verify_aggregate_seal_proof() (along with downstream logic)
- The rust-fil-proofs GitHub repository, commit a b958b5 on branch feat-aggregation, containing source code in filecoin-proofs/src/api/seal.rs with the primary functions aggregate_seal_commit _proofs() and verify_aggregate_seal_commit_p roofs() (along with downstream logic)
- The technical paper titled "Proofs for Inner Pairing Products and Applications" by Bünz et al.
- The private/preliminary technical paper titled "Proposal: Practical Groth16 Aggregation"¹
- Retest performed on bellperson Pull Request 179,² filecoin-ffi Pull Request 169³ and the technical paper "SNARKpack: Practical SNARK Aggregation" by Gailly, Maller and Nitulescu

The testing methodology revolved around documentation review and manual source code review augmented by fuzzing of selected components.

Limitations

While the target source code is part of a much larger out-of-scope system undergoing rapid development, NCC Group was able to achieve robust coverage of all in-scope components.

Key Findings

The in-scope code was well organized and supported by detailed technical documentation. Implementing the target functionality in Rust prevents many common memory-safety related errors, allows for safer use of concurrency and provides for a straightforward build/ test environment. Nonetheless, the review uncovered a total of eleven findings with the most notable including:

- Several instances of panic from malformed or malicious input that could present denial of service attack vectors or prevent a graceful recovery from errors.
- A known-constant value present in the random vector that determines the linear combinations of proof elements for the inner product process.
- The potential for memory churn and/or exhaustion during the deserialization of carefully crafted objects.
- Input validation checks utilizing debug_assertions that the compiler removes during a release build.

After retesting, NCC Group found **all findings were fixed, with one exception** of an informational observation involving toolchain and dependency versioning. This latter issue is expected to be fixed via a normal periodic updating process. Additional informational material is included as Appendix B on page 27.

Strategic Recommendations

Beyond addressing the reported findings, NCC Group recommends prioritizing the following areas during future development:

- Continue the focus on data validation for all input supplied at the API level and also whenever possible within intermediate functions (such as ensuring the lists provided to zip are of equal and non-zero lengths). Ensure range checks are implemented for both minimum and maximum expected values/sizes when appropriate.
- To better avoid denial of service scenarios stemming from unanticipated or illegal operations, prefer a R esult or Option over an assert which may panic at runtime, and over a debug_assert which will be removed during a release build. Continue utilizing the ensure!() macro, while avoiding unwrap() and expect() statements.
- Tighten the externally-visible API by reviewing the visibility of internal functions, perhaps replacing pub with pub(crate) when possible.

¹File Aggregate_Groth16_via_IPP.pdf dated March 11, 2021 with shasum ac40456...

²https://github.com/filecoin-project/bellperson/pull/179
³https://github.com/filecoin-project/filecoin-ffi/pull/169/commits/7f4effb29d1b0bd78125049c99da21839e778da3

Dashboard



Target Metadata		Engagement Data		
Name	Filecoin: Groth	6 Proof Aggregation	Туре	Cryptography and Implementation Review
Туре	Cryptographic I	_ibrary	Method	Manual Source Code Review, Fuzzing
Platforms	Rust with C FFI		Dates	2021-04-05 to 2021-04-16
Environment	Development		Consultants	3
			Level of Effort	15 person-days
Targets				
Bellperson	http	s://github.com/filecoin-	project/bellperson/	,
Filecoin-FFI	http	s://github.com/filecoin-	project/filecoin-ffi/	
Rust-Fil-Proofs	http	s://github.com/filecoin-	project/rust-fil-pro	ofs/
Finding Break	down			
Critical issues	0			
High issues	0			
Medium issues	5			
Low issues	4			
Informational iss	ues 2			
Total issues	11			
Category Brea	akdown			
Cryptography	5			
Data Validation	1			
Denial of Service	3			
Error Reporting	1			
Patching	1			
Component B	reakdown			
all	1			
	9			
bellperson				

Low

Informational

Key Critical High Medium

Table of Findings



For each finding, NCC Group uses a composite risk score that takes into account the severity of the risk, application's exposure and user population, technical difficulty of exploitation, and other factors. For an explanation of NCC Group's risk rating and finding categorization, see Appendix A on page 25.

Title	Status	ID	Risk
Constant Entry in Randomness Vector(s)	Fixed	003	Medium
Brittle Input Validation via Debug Assertions	Fixed	005	Medium
DoS in Aggregated Proof Verification via Malformed Proof	Fixed	006	Medium
Uncaught Panic in FFI Code	Fixed	007	Medium
Memory Exhaustion via Malformed Structured Reference String	Fixed	009	Medium
Potential DoS via Inverse Computation Panic	Fixed	002	Low
Panic when Aggregating Proofs of Length One	Fixed	004	Low
Aggregate Proof Malleability	Fixed	008	Low
Discrepancy between Reference and Implementation in KZG Challenge Point Computation	Fixed	011	Low
Marginally Inconsistent/Outdated Toolchain and Dependencies	Not Fixed	001	Informational
Missing Domain Separation Parameter in Oracle Calls	Fixed	010	Informational

Finding Details



Finding	Constant Entry in Randomness Vector(s)
Risk	Medium Impact: Medium, Exploitability: Low
Identifier	NCC-E001405-003
Status	Fixed
Category	Cryptography
Component	bellperson
Location	 structured_scalar_power() on lines 28-34 of bellperson/src/groth16/aggregate/ mod.rs Lines 49-57 of bellperson/src/groth16/aggregate/prove.rs Line 109 of bellperson/src/groth16/aggregate/verify.rs
Impact	A random r vector with the first element fixed to 1 (or r^0) will result in the first group elements of the Z_{AB} and Z_C inner product terms and the first v'_1 and v'_2 vector elements having no randomness applied. This impacts a central pillar of the aggregation scheme which depends upon checking a random linear combination of proof elements.
Description	The technical reference paper titled "Proposal: Practical Groth16 Aggregation" ⁴ section 3.3 contains an overview of the aggregation protocol stating "it is sufficient to prove that only one inner pairing product of a random linear combination of these initial equations defined by a verifier's random challenge $r \in \mathbb{Z}_p$ holds", with section 3.3.2 item 5 defining the randomness vector as $\mathbf{r} = \{r^i\}_{i=1}^n$. These statements are consistent with the approach articulated in the other technical paper "Proofs for Inner Pairing Products and Applications" ⁵ section 2.2.2 (although there are subsequent references to the first index and power starting at zero). The code comments on lines 51 and 53 below are aligned with the approach described above for the randomness vector and its inverse.
49 50 51 52 53 54 55 56 57	<pre>// Random linear combination of proofs let r = oracle!(&com_ab.0, &com_ab.1, &com_c.0, &com_c.1); // r, r^2, r^3, r^4 let r_vec = structured_scalar_power(proofs.len(), &r); // r^-1, r^-2, r^-3 let r_inv = r_vec .par_iter() .map(ri ri.inverse().unwrap()) .collect::<vec<_>>();</vec<_></pre>
	However, the implementation of structured_scalar_power() function in mod.rs excerpted below initializes the first element of the powers vector with a fixed value of F::one() on line 29.
28 29 30 31 32 33 34	<pre>fn structured_scalar_power<f: field="">(num: usize, s: &F) -> Vec<f> { let mut powers = vec![F::one()]; for i in 1num { powers.push(mul!(powers[i - 1], s)); } powers }</f></f:></pre>

⁴Aggregate_Groth16_via_IPP.pdf dated March 11, 2021 ⁵https://eprint.iacr.org/2019/1177.pdf



As a result, the first entries in the r_vec and r_inv vectors on lines 52 and 54 of the earlier code snippet are fixed and predictable without randomness. These vectors ultimately factor into subsequent calculations for the Z_{AB} and Z_C inner product terms, and the v'_1 and v'_2 vectors. While the test cases demonstrate that a constant value still works, this conflicts with the random linear combination pillar of the aggregation algorithm.

Recommendation In the structured_scalar_power() function, initiate the powers vector with s rather than F::one() on line 29.

Separately and informationally, since the intended length of the **powers** vector is known prior to its declaration, utilizing the Vec::with_capacity() initializer will eliminate repeated resizing which may marginally improve performance.

- **Retest Results** Regarding the Client Response noted immediately below, Protocol Labs offered the newly released paper titled "SnarkPack: Practical SNARK Aggregation" at https://eprint.iacr.org/20 21/529. Notably, the i index for constructing the randomness vector now starts at 0 in step 5 of the Prove algorithm described on page 13. This corresponds to the earlier external paper titled "Proofs for Inner Pairing Products and Applications" at https://eprint.iacr.org/2019/117 7.pdf which clearly indicates that the verifier constructs the randomness vector with a 1 in the leading position (which corresponds to *rⁱ* with an initial index of 0) on page 26. Pull Request 179 contained no code changes pertinent to this finding. While allotted time and project scope prevented NCC Group from performing a full analysis of the paper's implications, the code and technical documentation are now aligned here. As such, this finding has been marked 'Fixed'.
- Client Response Developer discussions over Slack greatly clarified the context to this finding. The constant value is proposed to be safe as-is and an internal writeup by Protocol Labs is being prepared to describe the supporting rationale. Additionally (lightly edited),

There is an ongoing PR which is bringing the paper and the implementation into "sync": namely, before we wrote the paper our scheme was using commitment keys as g^{a^i} and $g^{a^{n+1+i}}$ for i : [1, n]. We realized we could use the "smaller" bases g^{a^i} and $g^{a^{n+i}}$ later on for i : [0, n-1], so we wrote the paper using these bases, because it allows to aggregate more proofs. But the implementation was still using the old way. Because it led to some confusion, the PR is here to close that gap.



Finding	Brittle Input Validation via Debug Assertions
Risk	Medium Impact: Medium, Exploitability: Medium
Identifier	NCC-E001405-005
Status	Fixed
Category	Data Validation
Component	bellperson
Location	 Lines 13 and 30 of bellperson/src/groth16/aggregate/inner_product.rs Lines 78-81 of bellperson/src/groth16/aggregate/prove.rs Lines 288-291 of bellperson/src/groth16/aggregate/srs.rs
Impact	Depending upon the execution path, unvalidated values from external input may result in unexpected behavior and potential Denial of Service attacks.
Description	The pairing_miller_affine() function implemented in inner_product.rs returns the miller loop evaluated on pairs of inputs, as shown below. This function is usable externally as it appears to be marked pub from the crate root down to the function modifier ⁶ on line 12 shown below (though this may be a transient development artifact). Note that Rust's strict type checking ensures that proper parameter types are passed, and that the debug_assert _eq!() statement on line 13 validates the parameter contents.
12 13 14 15 16 17 18 19 20 21 22	<pre>pub fn pairing_miller_affine<e: engine="">(left: &[E::G1Affine], right: → &[E::G2Affine]) -> E::Fqk { debug_assert_eq!(left.len(), right.len()); let pairs = left .par_iter() .map(e e.prepare()) .zip(right.par_iter().map(e e.prepare())) .collect::<vec<_>>(); let pairs_ref: Vec<_> = pairs.iter().map((a, b) (a, b)).collect(); E::miller_loop(pairs_ref.iter()) }</vec<_></e:></pre>
	However, the Rust compiler eliminates debug assertions such as debug_assert_eq! when code is built for release, thus removing the input validation. When a test case was modified to invoke the above function with unequal length parameters and run via cargo testrelea se, the error was only caught when the test case compared actual versus expected results at its conclusion. This is somewhat fortuitous, as some functions such as copy_from_slice() ⁷ will panic on malformed input, which can lead to a denial of service. In this instance, if the length of left and right lists do not match, elements from the longer list will be considered extraneous and silently ignored by the zip function. See also finding NCC-E001405-006 on page 9 for another example of input validation and further perspective on consequences.

Note that the function cannot return any indication of failure, such as a Result⁸ or Option.

There are two additional instance of the same scenario present in the prove.rs and srs.rs source files.

⁶https://doc.rust-lang.org/reference/visibility-and-privacy.html ⁷https://doc.rust-lang.org/std/primitive.slice.html#method.copy_from_slice ⁸https://doc.rust-lang.org/std/result/



Recommendation	This finding is at the intersection of several concerns – function visibility, input validation and return types. Typically, a library should only expose the minimal API necessary for its use, each function in the exposed API should carefully validate all inputs, and the return type of each function should be able to indicate failure. In this specific instance:
	 Consider whether the function should be externally visible. The pub(crate) modifier may instead be appropriate.
	 Convert the debug_assert_eq!() statement to a functional non-debug equivalent, and consider whether additional input attributes can be validated.
	• Modify the function return type to Result or Option to support input validation failures.
	Note that there are other (out of scope) debug_assert statements present in the codebase
	in src/multicore.rs, src/groth16/proof.rs, src/groth16/multiscalar.rs and src/g
	roth16/verifier.rs.
Retest Results	A review of Pull Request 179 indicates the noted pairing_miller_affine() function now

has the pub(crate) visibility modifier, no longer contains a debug_assert and returns a Result<>. Separately, developer discussions indicate that the debug_assert in prove.rs is to support debugging rather than production. Similarly, the multiple debug_assert in srs.rs are contained within the setup_fake_srs() function which is also not a production path. As such, this finding was marked 'Fixed'.



Finding	DoS in Aggregated Proof Verification via Malformed Proof	
Risk	Medium Impact: Medium, Exploitability: Medium	
Identifier	NCC-E001405-006	
Status	Fixed	
Category	Denial of Service	
Component	bellperson	
Location	bellperson/src/groth16/aggregate/verify.rs:307	
Impact	By tampering with an aggregated proof, an attacker is able to trigger a crash during the verification process, effectively exercising a Denial of Service attack on the verifier.	
Description	Aggregated proofs are deeply nested structures composed of a large number of fields. Al proof types are declared in the bellperson/src/groth16/aggregate/proof.rs file. Specifically, the structure AggregateProof includes commitment values, aggregators, and a structure named TippMippProof. The latter contains KZG openings for the v and w values, as well as another proof structure called GipaProof. An adversary able to tamper with elements of this last GipaProof structure is able to perform a DoS on the verifier.	
	The representation of these different proofs is provided below, for reference.	
	<pre>pub struct AggregateProof<e: engine=""> { pub com_ab: commit::Output<e>, // pub tmipp: TippMippProof<e>, }</e></e></e:></pre>	
	<pre>pub struct TippMippProof<e: engine=""> { pub gipa: GipaProof<e>,</e></e:></pre>	
	<pre>pub vkey_opening: KZGOpening<e::g2affine>,</e::g2affine></pre>	
	<pre>pub wkey_opening: KZGOpening<e::g1affine>, }</e::g1affine></pre>	
	<pre>pub struct GipaProof<e: engine=""> { pub nproofs: u32,</e:></pre>	
	<pre>pub comms_ab: Vec<(commit::Output<e>, commit::Output<e>)>,</e></e></pre>	
	<pre>pub comms_c: Vec<(commit::Output<e>, commit::Output<e>)>,</e></e></pre>	
	<pre>pub z_ab: Vec<(E::Fqk, E::Fqk)>,</pre>	
	<pre>pub z_c: Vec<(E::G1, E::G1)>, pub final_a: E::G1Affine,</pre>	
	}	

When submitting an aggregated proof for verification with an empty vector for either tmip p.gipa.comms_ab, tmipp.gipa.comms_c, tmipp.gipa.z_ab, or tmipp.gipa.z_c, a crash is triggered in the function verify_tipp_mipp(), on line 207, where a call to the first()



function returns None, which is then unwrap()-ed and leads to a panic. An excerpt of that function is provided below for reference.



Note that the panic is triggered relatively deep into the call stack, which might indicate some oversights in the validations of parameters. Indeed, to verify an aggregated proof, one calls the verify_aggregate_proof() function, which in turns calls the gipa_verify_tipp_mip p() function (in which no error is triggered regarding the empty fields), before finally calling verify_tipp_mipp().

The reason for that crash is that the challenges vector returned by the gipa_verify_ti pp_mipp() function is empty. This is because the for-loop in that function (highlighted in the code excerpt below), tries to iterate over one of the empty vectors, and thus immediately stops, without triggering any error.

```
fn gipa_verify_tipp_mipp<E: Engine>(
    proof: &AggregateProof<E>,
) -> (GipaTUZ<E>, Vec<E::Fr>, Vec<E::Fr>) {
    info!("gipa verify TIPP");
    // ...
    // We first generate all challenges as this is the only consecutive process
    // that can not be parallelized then we scale the commitments in a
    // parallelized way
    for ((comm_ab, z_ab), (comm_c, z_c)) in comms_ab
        .iter()
        .zip(cs_ab.iter())
        .zip(comms_c.iter().zip(zs_c.iter()))
        {
    }
}
```

Recommendation Perform stricter parameter validation, as early as possible in the execution, and in particular



around functions that accept potential adversarial input. Additionally, consider writing functions that validate whether structures, such as proofs, are well-formed.

Retest Results Aparsing_check() method was added as part of Pull Request 173 (and also contained in Pull Request 179) to the AggregateProof structure, which ensures that proofs are well-formed.

This function is now called as a first step during the verification process performed by the verify_aggregate_proof() function, mitigating this finding.



Finding	Uncaught Panic in FFI Code	
Risk	Medium Impact: Medium, Exploitability: Medium	
Identifier	NCC-E001405-007	
Status	Fixed	
Category	Error Reporting	
Component	filecoin-ffi	
Location	fil_verify_aggregate_seal_proof() on lines 544-598 and convert_aggregation_inp uts() on lines 452-472 of filecoin-ffi/rust/src/proofs/api.rs	
Impact	An external caller to the Rust FFI function fil_verify_aggregate_seal_proof() may encounter a panic that does not gracefully return control (e.g., potentially a core dump).	
Description	The fil_verify_aggregate_seal_proof() function implemented on lines 544-598 of api. rs is called by an external FFI function to verify the output of an aggregated seal. The function returns a pointer to a constructed fil_VerifyAggregateSealProofResponse struct defined in filecoin-ffi/rust/src/proofs/types.rs which includes an is_valid boolean field. After some initial input validation, the function calls convert_aggregation_inputs() on each element of a slice of the commit_inputs.	
	The convert_aggregation_inputs() function is implemented on lines 452-472 of api.rs contains a statement involving .unwrap_or_else(_ { panic!(When this is encountered control may not be returned to the caller in a graceful manner.	
	Note that other nearby functions implement a similar pattern surrounded by a catch_unwind function. ⁹ Rust documentation indicates that:	
	It is currently undefined behavior to unwind from Rust code into foreign code, so this function is particularly useful when Rust is called from another language (normally C). This can run arbitrary Rust code, capturing a panic and allowing a graceful handling of the error.	
	It is not recommended to use this function for a general try/catch mechanism. The Result type is more appropriate to use for functions that can fail on a regular basis. Additionally, this function is not guaranteed to catch all panics, see the "Notes" section below.	
	However, the catch_unwind() function is absent from the noted location. Further, lines 580- 592 indicate functionality that does involve the is_valid boolean flag (and is thus unused for this panic case).	
Recommendation	Indicate function failure through the Result or Option mechanism (e.g., through the is_v alid flag) rather than through a panic. The catch_unwind mechanism can be a secondary 'defense in depth' mechanism, but should not be the primary intended functional path.	
Retest Results	A review of Pull Request 169 indicates the convert_aggregation_inputs() function no longer panics but rather returns a Result per the recommendation. Additionally, the primary return path for the fil_aggregate_seal_proofs() function now involves a Result<> and also contains a catch_panic_response mechanism (with catch_unwind) similar to its siblings. As such, this finding has been marked 'Fixed'.	
	⁹ https://doc.rust-lang.org/std/panic/fn.catch_unwind.html	



Finding	Memory Exhaustion via Malformed Structured Reference String
Risk	Medium Impact: High, Exploitability: Medium
Identifier	NCC-E001405-009
Status	Fixed
Category	Denial of Service
Component	bellperson
Location	bellperson/src/groth16/aggregate/srs.rs:345bellperson/src/groth16/aggregate/srs.rs:216
Impact	An adversary may trigger the allocation of large amounts of memory, eventually impeding the normal behavior of processes.
Description	In the file srs.rs, the functions read() and read_mmap() are used to parse a Structured Reference String (SRS). Both functions use helper functions to read vectors (read_vec() and mmap_read_vec(), respectively), and follow the same pattern when doing so; they first read a 32-bit integer, and then allocate a vector of that length. This can be seen in the highlighted portion of the read_vec() function provided below for reference.
	<pre>fn read_vec<g: curveaffine,="" r:="" read="">(r: &mut R) -> io::Result<vec<g>> { let vector_len = r.read_u32::<bigendian>()? as usize; let mut data = vec![G::Compressed::empty(); vector_len]; for encoded in &mut data { r.read_exact(encoded.as_mut())?; } Ok(data .par_iter() .map(lencl { enc.into_affine() .map_err(lel io::Error::new(io::ErrorKind::InvalidData, e)) .and_then(lsl Ok(s)) }) .collect::<io::result<vec<_>>>()?) }</io::result<vec<_></bigendian></vec<g></g:></pre>
	In the case of a malformed (potentially adversarial) SRS, a large amount of memory may be incorrectly allocated, which would impede normal functioning. Specifically, a 5-byte vector may result in the allocation of more than 4GB of memory.
Recommendation	Consider enforcing an upper bound on the expected size of SRS fields when parsing them. Additionally, check possible mismatches between the parsed lengths and expected buffer sizes (possibly also enforcing 0 length checks) where appropriate.
Retest Results	A MAX_SRS_SIZE constant was added as part of Pull Request 173 (and also contained in Pull Request 179) to the bellperson/src/groth16/aggregate/srs.rs file. When reading data using the functions read_vec() and mmap_read_vec(), an error is now returned if the vector length is larger than this upper bound, mitigating this finding.



Finding	Potential DoS via Inverse Computation Panic
Risk	Low Impact: Medium, Exploitability: Low
Identifier	NCC-E001405-002
Status	Fixed
Category	Cryptography
Component	bellperson
Location	bellperson/src/groth16/aggregate/verify.rs:73
Impact	With overwhelmingly low probability, a panic may be triggered during the aggregated proof verification process, effectively exercising a Denial of Service attack on the verifier. This condition may also be leveraged by an attacker.
Description	The function verify_aggregate_proof() is the main function used to verify aggregated proofs. During the verification process, a random challenge r is computed and then used to check the aggregate pairing product equation. During this computation, one is subtracted to r before computing the inverse of that quantity. The code excerpted below highlights this process.
	<pre>// Check aggregate pairing product equation // SUM of a geometric progression // SUM a^i = (1 - a^n) / (1 - a) = -(1-a^n)/-(1-a) // = (a^n - 1) / (a - 1) info!("checking aggregate pairing"); let mut r_sum = r.pow(&[public_inputs.len() as u64]); r_sum.sub_assign(&E::Fr::one()); let b = sub!(r, &E::Fr::one()).inverse().unwrap(); r_sum.mul_assign(&b);</pre>
	If the variable r were equal to one, the result of the subtraction would be 0, which does not have a modular inverse in the field. Thus, the call to inverse() above would return None , which would trigger a panic when unwrap() -ing. However, since that variable is computed as the output of a hash function (which is used to model a random oracle based on the Fiat-Shamir heuristic, see below), the probability of it being one is negligible.
	<pre>// Random linear combination of proofs let r = oracle!(&proof.com_ab.0, &proof.com_ab.1,</pre>

&proof.com_ab.1, &proof.com_c.0, &proof.com_c.1

);

Note that the input to the random oracle may be controlled by an adversary. However, it does not seem feasible for an attacker to craft inputs hashing to such value.

The NCC Group team also noted that the implementation of the oracle currently guards against another such edge case. Specifically, if the value computed as the output of the hash function does not have an inverse, the oracle will perform additional iterations in order to generate a value for which an inverse exists.

Recommendation Consider adding a check in the oracle macro to protect against the generation of one.



Retest Results	As part of Pull Request 173 (and also contained in Pull Request 179), the following check that mitigates this finding was added to bellperson/src/groth16/aggregate/macros.rs:
	<pre>if c == one { continue; }</pre>



Finding	Panic when Aggregating Proofs of Length One	
Risk	Low Impact: Medium, Exploitability: Medium	
Identifier	NCC-E001405-004	
Status	Fixed	
Category	Denial of Service	
Component	bellperson	
Location	bellperson/src/groth16/aggregate/prove.rs:109	
Impact	A panic may be triggered during the proof aggregation process, effectively exercising a Denial of Service attack on the prover.	
Description	The proof aggregation process, implemented in the aggregate_proofs() function, converses an array of n zkSNARK proofs to an aggregate proof. Currently, the number of aggregate proofs must be a power of 2. The aggregate_proofs() function starts by performing a sanity checks on its inputs before proceeding with the aggregation, as can be seen in the converse below.	
	<pre>/// Aggregate `n` zkSnark proofs, where `n` must be a power of two. pub fn aggregate_proofs(E: Engine + std::fmt::Debug>(srs: &ProverSRS(E), proofs: &[Proof {E}],) -> Result<aggregateproof synthesiserror="" {e},=""> { if !proofs.len().is_power_of_two() { return Err(SynthesisError::NonPowerOfTwo); } if !srs.has_correct_len(proofs.len()) { return Err(SynthesisError::MalformedSrs); } // // Random linear combination of proofs let r = oracle!(&com_ab.0, &com_ab.1, &com_c.0, &com_c.1); // r, r^2, r^3, r^4 let r_vec = structured_scalar_power(proofs.len(), &r); // r^-1, r^-2, r^-3 // // we prove tipp and mipp using the same recursive loop let proof = prove_tipp_mipp:::<e>(&srs, &a, &b_r, &c, &wkey_r_inv, &r_vec)?; Howaver if the number of individual zkSNAPK proofs passed in to the aggregate proof </e></aggregateproof></pre>	

However, if the number of individual zkSNARK proofs passed in to the aggregate_proof s() function is equal to 1, a panic will be triggered in the prove_tipp_mipp() function, highlighted above. Note that 1 is a power of two ($2^0 = 1$), and as such the check proofs .len().is_power_of_two() above will succeed.

More specifically, the prove_tipp_mipp() function tries to access the r_vec array at index 1, which is the same length as the proof array, resulting in an 'index out of bounds' panic if the latter is of length one. This is highlighted in the code excerpt below.



```
fn prove_tipp_mipp<E: Engine>(
                         srs: &ProverSRS <E>,
                         a: &[E::G1Affine],
                         b: &[E::G2Affine],
                         c: &[E::G1Affine],
                         wkey: &WKey<E>, // scaled key w^r-1
                         r_vec: &[E::Fr],
                     ) -> Result<TippMippProof<E>, SynthesisError> {
                         if !a.len().is_power_of_two() || a.len() != b.len() {
                             return Err(SynthesisError::MalformedProofs);
                         let r_shift = r_vec[1].clone();
Reproduction Steps Change the NUM_PROOFS value to 1 in the test test_groth16_aggregation() of the file be1
                    lperson/tests/groth16_aggregation.rs and run the test to observe the panic.
                     const NUM_PROOFS: usize = 1;
 Recommendation Consider adding a guard to prevent aggregation of less than 2 proofs. Additionally, consider
                    performing stricter parameter validation, as early as possible in the function executions.
     Retest Results As part of Pull Request 173 (and also contained in Pull Request 179), the following check was
                    added to the function aggregate_proofs() in bellperson/src/groth16/aggregate/pro
                    ve.rs:
                     if proofs.len() < 2 {</pre>
                         return Err(SynthesisError::MalformedProofs(
                              "aggregating less than 2 proofs is not allowed".to_string(),
                         ));
                    This check now mitigates the finding.
```



Finding	Aggregate Proof Malleability
Risk	Low Impact: Low, Exploitability: Low
Identifier	NCC-E001405-008
Status	Fixed
Category	Cryptography
Component	bellperson
Location	bellperson/src/groth16/aggregate/verify.rs:307
Impact	An adversary may be able to arbitrarily inflate aggregated proofs. If other components were relying on assumptions surrounding proof non-malleability, unexpected issues might occur.
Description	Aggregated proofs are deeply nested structures composed of a large number of fields. All proof types are declared in the bellperson/src/groth16/aggregate/proof.rs file. Specifically, the structure AggregateProof includes commitment values, aggregators, and a structure named TippMippProof. The latter contains KZG openings for the v and w values, as well as another proof structure called GipaProof. A sibling finding (see finding NCC-E001405-006 on page 9) shows the nesting between these proofs and provides code excerpts.
	An adversary able to tamper with elements of this GipaProof structure (given a valid instance of an AggregateProof) may arbitrarily inflate the proof, and the verification will still succeed. Note that this <i>does not mean</i> that an adversary is able to forge individual (or aggregated) zkSNARK proofs.
	Similar to the sibling finding referred to above, the reason for the malleability of this proof structure is twofold: the verify_aggregate_proof() function performs limited parameter checks, and the Rust zip() operator on iterators will return as soon as one of the iterators is exhausted. An excerpt of vulnerable code from the verify_aggregate_proof() function is provided below.
	<pre>for ((comm_ab, z_ab), (comm_c, z_c)) in comms_ab .iter() .zip(zs_ab.iter()) .zip(comms_c.iter().zip(zs_c.iter())) { //</pre>
	As a result, when submitting an aggregated proof for verification with vectors of different sizes for either tmipp.gipa.comms_ab, tmipp.gipa.comms_c, tmipp.gipa.z_ab, or tmipp .gipa.z_c, the function will loop over elements of these fields until it reaches the end of the shortest vector, at which point it will exit. Thus, an attacker may inflate the proof structure by appending an arbitrary number of elements to either of these fields, and the verification will succeed.
	The exploitability (and corresponding risk rating) of this finding was set to <i>Low</i> , since the function to read an AggregateProof performs appropriate checks on the fields of the proof

function to read an AggregateProof performs appropriate checks on the fields of the proof structures and the function verify_aggregate_proof() is not expected to be publicly accessible, as per the Filecoin team.

Reproduction Steps In bellperson/tests/groth16_aggregation.rs, add the following lines to the test_groth16_aggregation() and observe that the test still succeeds.



Retest Results A parsing_check() method was added as part of Pull Request 173 (and also contained in Pull Request 179) to the AggregateProof structure, which ensures that proofs are well-formed.

This function is now called as a first step during the verification process performed by the verify_aggregate_proof() function, mitigating this finding.



Finding	Discrepancy between Reference and Implementation in KZG Challenge Point Com- putation	
Risk	Low Impact: Low, Exploitability: Medium	
Identifier	NCC-E001405-011	
Status	Fixed	
Category	Cryptography	
Component	bellperson	
Location	bellperson/src/groth16/aggregate/prove.rs:124bellperson/src/groth16/aggregate/verify.rs:196	
Impact	Discrepancies between reference and implementation may invalidate security proofs, result- ing in potential flaws in the protocol.	
Description	During the proof aggregation process, commitment to a polynomial using KZG is performed, which utilizes the output of a random oracle as a challenge point.	
	The computation of the challenge is performed during the proof aggregation process in prove.rs, on line 123 (see the excerpt below), and in the proof verification process in the file verify.rs, on line 195.	
	<pre>// KZG challenge point let z = oracle!(&challenges[0], &proof.final_vkey.0, &proof.final_vkey.1, &proof.final_wkey.0, &proof.final_wkey.1);</pre>	
	This computation differs from the (internal) reference paper "Proposal: Practical Groth16 Aggregation" in two ways.	
	First, the paper states that the KZG challenge point is computed with all the challenges x (in bold, referring to a vector). Namely, towards the end of the MIPP . Prove procedure, at the top of p. 20, it states:	
	Draw challenge $z = Hash_2(\mathbf{x},v1,v2)$ (from all challenges \mathbf{x} and $(v1,v2)$)	

However, the implementation uses a single challenge, the first element of the challenge s vector (as can be seen above), instead of using the vector containing all the challenges computed. While the element used in the point computation *does* incorporate all the previous challenges (since it is iteratively computed as a hash of the previous challenge), this does not strictly follow the specification.

Second, the computation of the KZG challenge point is performed in two instances in the paper; once during the TIPP proof and once during the MIPP proof. The inputs to the hash function differ for these two proofs. Consider the **TIPP**.**Prove** procedure, in which the challenge point is computed as follows (as can be seen on p. 22):

Draw challenge $z = Hash_2(\mathbf{x}, v1, v2, w1, w2)$



	This differs from the computation performed in MIPP. Prove shown above, with the additional inclusion of the commitment keys $w1, w2$. However, the implementation uses the same challenge computation for both proofs, namely the one performed in the TIPP.Prove procedure. It is unclear whether this has a security impact.
	Finally, the NCC Group team noted that the paper uses a number of different notations for the computation of this challenge point. In addition to the two listed above, the following two notations appear in the paper.
	On p. 20:
	1. Reconstruct KZG challenge point: $z=H(x_{log(n)},v1,v2)$
	On p. 22:
	1. Reconstruct KZG challenge point: $z = H(\{x_i\}_{i=0}^{log(n)}, v1, v2, w', w')$
Recommendation	Ensure that the reference paper and the implementation match, and that notation is consis- tent throughout the paper.
	If possible, consider providing an argument (in the implementation or the reference paper) supporting the use of a single KZG challenge point for both proofs.
Retest Results	Pull Request 172 (which is also contained in Pull Request 179) combines a number of changes aiming to synchronize the implementation with the reference paper.
	Instead of addressing the discrepancies listed in this finding, the reference paper ¹⁰ was up- dated in the following way.
	 Some arguments supporting the security of the combination of the TIPP and MIPP proofs and of their respective challenges were included. The challenge KZG point is now computed using the last challenge only.
	This addresses the concerns listed in this finding.

¹⁰https://eprint.iacr.org/2021/529



Finding	Marginally Inconsistent/Outdated Toolchain and Dependencies
Risk	Informational Impact: Undetermined, Exploitability: Undetermined
Identifier	NCC-E001405-001
Status	Not Fixed
Category	Patching
Component	all
Location	 bellperson/rust-toolchain bellperson/Cargo.toml filecoin-ffi/rust/rust-toolchain rust-fil-proofs/Cargo.toml rust-fil-proofs/Cargo.toml
Impact	Attackers may attempt to identify and utilize publicly known vulnerabilities in outdated depen- dencies to exploit the functionality of the target code.
Description	Incorporating outdated dependencies is one of the most common, serious and exploited application vulnerabilities. Inconsistent toolchains can also increase the difficulty of build and debug.
	The bellperson repository contains a rust-toolchain file set to stable version 1.46.0, which is about 6-months out of date. ¹¹ The filecoin-ffi repository contains a rust-toolchain file set to nightly-2020-10-5 of similar vintage. Note that the rust-fil-proofs repository contains a rust-toolchain file set to stable version 1.51.0 which is fully up to date. These are inconsistent and the former two are outdated.
	Each repository includes a Cargo.tom1 file specifying dependencies. Some dependencies are marginally out of date, including:
	 bellperson ff specifies v0.2.0, while the latest fff version is v0.2.3 rust-gpu-tools specifies v0.2.0, while the latest version is v0.3.0 rand_core specifies v0.5, while the latest version is v0.6.2 byteorder specifies v1. while the latest version is v1.4.3 rand specifies v1.3.0, while the latest version is v1.5.0 itertools specifies v0.9.0, while the latest version is v0.10.0 filecoin-ffi bls-signatures specifies v0.8, while the latest version is v0.9.0 ff specifies v0.2.1, while the latest fff is v0.2.3 rust-gpu-tools specifies v0.2.0, while the latest version is v0.3.0
	In addition, the Cargo.tom1 files make use of the [patch.crates-io] directive which fur- ther increases the challenge of version management, since branches rather than commits, versions or digests are specified.
Recommendation	Update the rust-toolchain files to the (same) latest stable version recommended for pro- duction deployment, which is currently 1.51.0. If a repository requires the nightly channel, use a version of the same vintage. Update the Cargo.toml files to include the most recent
	¹¹ https://github.com/rust-lang/rust/blob/master/RELEASES.md#version-1460-2020-08-27



versions of dependencies and minimize the complexity of [patch.crates-io] clauses.

Retest Results A review of Pull Request 179 does not indicate changes relevant to the rust-toolchain or Cargo.toml files nor the dependency versions. Note that this informational observation does not indicate an immediate security issue. The developers indicate this will be resolved through the regular update process. As such, this finding has been marked 'Not Fixed' at this time.



Finding	Missing Domain Separation Parameter in Oracle Calls
Risk	Informational Impact: Undetermined, Exploitability: Low
Identifier	NCC-E001405-010
Status	Fixed
Category	Cryptography
Component	bellperson
Location	 bellperson/src/groth16/aggregate/prove.rs:124 bellperson/src/groth16/aggregate/verify.rs:196 bellperson/src/groth16/aggregate/prove.rs:233 bellperson/src/groth16/aggregate/verify.rs:318
Impact	Calls to different random oracles may result in the same output, contradicting the random oracle model on which the security proofs are built.
Description	When implementing cryptography protocols that were proven in the Random Oracle Model, it is common practice to instantiate random oracles using hash functions. Generally, when using a single hash function to model multiple random oracles, the hash functions are instantiated using different <i>domain separators</i> . This ensures that calls to different oracles with the same inputs will result in different outputs, so as not to invalidate the assumptions made in the security proof.
	The (internal) reference paper "Proposal: Practical Groth16 Aggregation" describes two different hash functions used in the course of the proof aggregation and verification, $Hash_1$ and $Hash_2$, which are used to derive challenges for the proof commitments.
	In the implementation, both hash functions are implemented by the oracle macro, in bellp erson/src/groth16/aggregate/macros.rs, which hashes all its inputs using SHA256.
	The NCC Group team noticed that no domain separator was passed in the oracle calls. As such, the two hash functions upon which the proofs are built are effectively the same, possibly invalidating the security proof. An example present in the verify.rs file is excerpted below.
	<pre>let c = oracle!(&challenges.first().unwrap(), &fvkey.0, &fvkey.1, &&fwkey.0, &&fwkey.1);</pre>
Recommendation	Consider adding a domain separator tag to the oracle calls, different for the $Hash_1$ and $Hash_2$ calls.
Retest Results	As part of Pull Request 172 (and also contained in Pull Request 179), two different domain separation tags ("randomr" and "randomgipa", respectively) were added to the oracle calls, addressing this finding.



The following sections describe the risk rating and category assigned to issues NCC Group identified.

Risk Scale

NCC Group uses a composite risk score that takes into account the severity of the risk, application's exposure and user population, technical difficulty of exploitation, and other factors. The risk rating is NCC Group's recommended prioritization for addressing findings. Every organization has a different risk sensitivity, so to some extent these recommendations are more relative than absolute guidelines.

Overall Risk

Overall risk reflects NCC Group's estimation of the risk that a finding poses to the target system or systems. It takes into account the impact of the finding, the difficulty of exploitation, and any other relevant factors.

- **Critical** Implies an immediate, easily accessible threat of total compromise.
- **High** Implies an immediate threat of system compromise, or an easily accessible threat of large-scale breach.
- **Medium** A difficult to exploit threat of large-scale breach, or easy compromise of a small portion of the application.
 - Low Implies a relatively minor threat to the application.
- **Informational** No immediate threat to the application. May provide suggestions for application improvement, functional issues with the application, or conditions that could later lead to an exploitable finding.

Impact

Impact reflects the effects that successful exploitation has upon the target system or systems. It takes into account potential losses of confidentiality, integrity and availability, as well as potential reputational losses.

- **High** Attackers can read or modify all data in a system, execute arbitrary code on the system, or escalate their privileges to superuser level.
- **Medium** Attackers can read or modify some unauthorized data on a system, deny access to that system, or gain significant internal technical information.
 - **Low** Attackers can gain small amounts of unauthorized information or slightly degrade system performance. May have a negative public perception of security.

Exploitability

Exploitability reflects the ease with which attackers may exploit a finding. It takes into account the level of access required, availability of exploitation information, requirements relating to social engineering, race conditions, brute forcing, etc, and other impediments to exploitation.

- **High** Attackers can unilaterally exploit the finding without special permissions or significant roadblocks.
- **Medium** Attackers would need to leverage a third party, gain non-public information, exploit a race condition, already have privileged access, or otherwise overcome moderate hurdles in order to exploit the finding.
 - **Low** Exploitation requires implausible social engineering, a difficult race condition, guessing difficult-toguess data, or is otherwise unlikely.



Category

NCC Group categorizes findings based on the security area to which those findings belong. This can help organizations identify gaps in secure development, deployment, patching, etc.

Access Controls	Related to authorization of users, and assessment of rights.
Auditing and Logging	Related to auditing of actions, or logging of problems.
Authentication	Related to the identification of users.
Configuration	Related to security configurations of servers, devices, or software.
Cryptography	Related to mathematical protections for data.
Data Exposure	Related to unintended exposure of sensitive information.
Data Validation	Related to improper reliance on the structure or values of data.
Denial of Service	Related to causing system failure.
Error Reporting	Related to the reporting of error conditions in a secure fashion.
Patching	Related to keeping software up to date.
Session Management	Related to the identification of authenticated users.
Timing	Related to race conditions, locking, or order of operations.

Appendix B: Engagement Notes and Observations **NCC**ဝ(ဂပပ

This informational section highlights selected portions of the engagement methodology used, some of the priority concerns investigated, and observations that do not warrant security-related findings on their own. The primary strategy for this project relied heavily on manual source code inspection, supported by some execution of the included test cases and light fuzzing of some of the parsing functions. Priority was given to the correctness of cryptographic algorithms and implementation, the specific focus areas highlighted in the executive summary, data validation, control flow and general secure coding practices that could potentially impact legitimate operation.

Build Environment

The team evaluated the build environment (which resulted in finding NCC-E001405-001 on page 22) and ran the tool cargo audit on the three repositories under review. Two vulnerabilities were discovered: one crate used by the three repositories (fil-ocl) was found to be vulnerable, while another crate (raw-cpuid) used by rust-fil-proo fs was also vulnerable. In addition, the cargo audit tool highlighted several other deprecated crates in the three repositories. The latter item shown below is addressed as part of the noted finding's recommendation.

ID:	RUSTSEC-2021-0011	
Crate:	fil-ocl	
Version:	0.19.6	
Date:	2021-01-04	
URL:	https://rustsec.org/advisories/RUSTSEC-2021-0011	
Title:	EventList's From <eventlist> conversions can double drop on panic.</eventlist>	
Solution:	No safe upgrade is available!	
Dependenc	y tree:	
fil-ocl 0.19.6		
-− rust-	gpu-tools 0.3.0	
L rust-	gpu-tools 0.2.2	
ID:	RUSTSEC-2021-0013	
Crate:	raw-cpuid	
Version:	8.1.2	
Date:	2021-01-20	
URL :	https://rustsec.org/advisories/RUSTSEC-2021-0013	
Title:	Soundness issues in `raw-cpuid`	
Solution:	upgrade to $>=$ 9.0.0	
Dependenc	y tree:	
raw-cpuid	8.1.2	
└── fil-proofs-tooling 5.5.0		

Fuzzing

The project supports two different backends for field, curve and pairing operations, namely the **blst** and **paired** crates. The integration with external dependencies introduces the potential for serialization/deserialization bugs. NCC Group built and ran custom fuzzers on the following blocks for both backends:

- crate::bellperson::groth16::aggregate::AggregateProof::<Bls12>::read()¹²
- crate::bellperson::groth16::aggregate::GipaProof::<Bls12>::read()¹³
- crate::bellperson::groth16::aggregate::TippMippProof::<Bls12>::read()¹⁴

One finding arose from this effort (see finding NCC-E001405-009 on page 13), while a second finding (see finding NCC-E001405-005 on page 7) was stimulated by the fuzzing development process.

Comparison between Reference Paper and Implementation

The two technical references were used as a support to assess the implementation. Some discrepancies with limited impact were identified and are discussed in more details in finding NCC-E001405-003 on page 5 and finding NCC-

 ¹²https://github.com/filecoin-project/bellperson/blob/d4120782d27e2971bd19ac5e12d206a7ccffbb7a/src/groth16/aggregate/proof.rs#L82
 ¹³https://github.com/filecoin-project/bellperson/blob/d4120782d27e2971bd19ac5e12d206a7ccffbb7a/src/groth16/aggregate/proof.rs#L233
 ¹⁴https://github.com/filecoin-project/bellperson/blob/d4120782d27e2971bd19ac5e12d206a7ccffbb7a/src/groth16/aggregate/proof.rs#L355



E001405-011 on page 20.

The team paid particular attention to the good generation and use of randomness, which is especially important to ensure the security of the aggregate proof verification. Indeed, the aggregate proof verification process checks a single randomized equation, which, if it were not random, might be exploited by an attacker.

The generation of a Structured Reference String (SRS) is typically a very sensitive operation; in other protocols such as the ZCash powers of tau ceremony, extensive care¹⁵ was taken to erase the toxic waste. Since the proof aggregation and verification make use of two existing SRSs, the need for memory zeroization in that regard is not necessary.

Input Validation

Input validation was inspected manually, by debugger-introduced variations, and by modified test cases where possible. Initial focus was placed on field range and curve subgroup validation, such as the deserialization and appropriate range check of Fq, or the correct subgroup check on G1Projective during deserialization.

The team noted a number of instances that could benefit from better input validation, as evidenced by findings such as finding NCC-E001405-006 on page 9 and finding NCC-E001405-004 on page 16. In addition, in the spirit of defense in depth, it might be beneficial to include such checks throughout the code, even if the functions do not seem susceptible to be used externally. For example, in bellperson/src/groth16/aggregate/mod.rs, in the compress() function (provided below for reference), if the value of split were larger than the length of the vector, the assertion in split_at_mut() would fail.

```
fn compress<C: CurveAffine>(vec: &mut Vec<C>, split: usize, scaler: &C::Scalar) {
    let (left, right) = vec.split_at_mut(split);
    // ...

pub fn split_at_mut(&mut self, mid: usize) -> (&mut [T], &mut [T]) {
    assert!(mid <= self.len());
</pre>
```

A second example from the **rust-fil-proofs** repository source file **seal.rs**:801 is excerpted below. The function **verify_aggregate_seal_commit_proofs()** performs an unchecked division via the remainder operator % on an unvalidated caller-provided function parameter. If **aggregated_proofs_1en** is zero, this will result in a **panic**.

```
810 ensure!(
811 commit_inputs.len() % aggregated_proofs_len == 0,
812 "invalid number of inputs provided"
813 );
```

Rust Programming Practices

The team noted a few instances of potentially unsafe Rust programming practices, for example in the use of unwrap() (such as in finding NCC-E001405-002 on page 14). Generally speaking, explicit error handling should be preferred instead of calling functions that might result in panics, such as unwrap() or expect(). The Secure Rust Guidelines provide some helpful pointers to that effect.

As an example, the function <code>aggregate_seal_commit_proofs()</code> contains two instances of <code>.expect()</code> shown below, which will cause a panic if triggered. The same function (and others in this file) use the preferred <code>ensure!()</code> macro in other places to pass error messages rather than panic.

```
726 let mut proofs: Vec<_> = commit_outputs
727 .iter()
728 .fold(Vec::new(), |mut acc, commit_output| {
```

¹⁵https://z.cash/technology/paramgen/



729	acc.extend(
730	MultiProof::new_from_reader(
731	Some(partitions),
732	&commit_output.proof[],
733	&verifying_key,
734)
735	<pre>.expect("failed to construct multi proof from bytes")</pre>
736	.circuit_proofs,
737);
738	
739	acc
740	});
760	proofs.push(
761	proofs
762	.last()
763	<pre>.expect("failed to access last proof for duplication")</pre>

Adapting the above logic to utilize the preferred **ensure!()** macro will increase the ability for callers to gracefully handle errors.

Separately, another particular item of note is the use of the zip() operator to iterate through multiple vectors at the same time. Together with the lack of confirmation of equal-length lists prior to zip-ing them, this has shown to be a potential vector of issues (see finding NCC-E001405-006 on page 9 and finding NCC-E001405-008 on page 18) that could be avoided by adding an inexpensive check that will prevent an "orphan" from being ignored (and potentially enabling an escape). There are many instances where this operator is used. For example (as also noted in finding NCC-E001405-005 on page 7), the following computation in bellperson/src/groth16/aggregate/inner_product.rs might return incorrect results in non-debug builds.

```
pub fn pairing_miller_affine<E: Engine>(left: &[E::G1Affine], right: &[E::G2Affine]) -> E::Fqk {
    debug_assert_eq!(left.len(), right.len());
    let pairs = left
        .par_iter()
        .map(|e| e.prepare())
        .zip(right.par_iter().map(|e| e.prepare()))
        .collect::<Vec<_>>();
    let pairs_ref: Vec<_> = pairs.iter().map(|(a, b)| (a, b)).collect();
    E::miller_loop(pairs_ref.iter())
}
```

Optimization Potential

.clone(),

764

765);

Generally, the cost of performing a field inversion is significantly larger than the cost of computing a field multiplication. In **bellperson/src/groth16/aggregate/prove.rs**, the inverses of the powers of *r* are computed, as can be seen in the code excerpt below.

```
// r, r^2, r^3, r^4 ...
let r_vec = structured_scalar_power(proofs.len(), &r);
// r^-1, r^-2, r^-3
let r_inv = r_vec
    .par_iter()
    .map(|ri| ri.inverse().unwrap())
    .collect::<Vec<_>>();
```



Thus, for an array of length n, the code above computes n modular inversions. A possible optimization to this piece of code might be to compute r^{-n} (only one inversion) and then successively multiply it by r, thus obtaining $r^{-n+1}, \ldots, r^{-2}, r^{-1}$.

Additionally, there are instances of iteratively building long vectors where the length is known in advance. Declaring new vectors with Vec::with_capacity¹⁶ may significantly reduce the otherwise repeated memory allocation overhead.

¹⁶https://doc.rust-lang.org/std/vec/struct.Vec.html#capacity-and-reallocation